MODEL SAMPLE PAPER I - 2011

MATHEMATICS-XII

Time allowed: 3 hours **Max Marks: 100**

General Instructions:

a. This question paper consists of 29 questions divided into three sections A, B &C.
b. Section A consists of 10 questions each of 1 mark.
c. Section B consists of 12 questions each of 4 marks.
d. Section B consists of 7 questions each of 6 marks.
e. All questions are compulsory.

SECTION - A

1. Let $*$ be a binary operations defined by a $*$ b = 3a + 4b – 2. Find 2 $*$ 3.

2. If sin [$\tan^{-1}\frac{1}{5} + \cot^{-1} x$] = 1, find the value of x.

3. Let A = $\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$ and B = [[illegible] 2 3], find AB.

4. If $\begin{bmatrix} x+3 & 4 \\ y-4 & x+y \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 3 & 9 \end{bmatrix}$, find x and y.

5. If $\begin{bmatrix} x+2 & 3 \\ x+5 & 4 \end{bmatrix}$ is a singular matrix find the value of x.

6. Evaluate: $\int_{-\pi/2}^{\pi/2} \frac{x}{1+\sin^2 x}\,dx$.

7. Evaluate: $\int e^x (\sin x + \cos x)\,dx$.

8. If $|\vec{a} \cdot \vec{b}| = 6$ and $|\vec{a}| = 3$, find the projection of $\vec{b}$ on $\vec{a}$.

9. Let $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} + \hat{k}$, find the unit vector in the direction of $\vec{b} + \vec{a}$.

10. Find p such that $\frac{x}{1} = \frac{y}{2} = \frac{z}{3p}$ and $\frac{x}{-2} = \frac{y}{4} = \frac{z}{1}$ are perpendicular to each other.

SECTION - B

11. Consider f : R+→(-5, ∞) given by f(x) = 9 x^2 + 6x -5. Show that f is invertible with

$$f^{-1}(y) = \frac{\sqrt{y+6}-1}{3}$$

12. Solve for x : $2\tan^{-1} x = \sin^{-1}\left(\frac{2a}{1+a^2}\right) + \cos^{-1}\left(\frac{1-b^2}{1+b^2}\right)$

13. Prove the following by Principle of mathematical Induction, if A = $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ $then, A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$

for every integer n.

OR

Using properties of the determinants prove that: $\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix}$ = $4a^2b^2c^2$.

14. Determine the value of 'a' so that the following function is continuous.

f(x) = $\begin{cases} ax^2 + b, & x \rangle 2 \\ 2, & x = 2 \\ 2ax - b, & x \langle 2 \end{cases}$

15. Differentiate, $(\sin x)^x + \tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$

OR

If x = 3 sin t – sin 3t, y = 3 cos t + cos 3t, find $\frac{d^2y}{dx^2}\, at\, t = \frac{\pi}{3}$

16. Find the intervals in which the function f(x) = $2x^3 - 9x^2 - 24x - 5$ is increasing or decreasing.

17. Evaluate: $\int \frac{dx}{x(x^n + 1)}$

OR

Evaluate: $\int \frac{x+1}{(x+3)^3} e^x \, dx$

18. Form the differential equation of the family of curves y = a e^{2x} + b e^{-2x} by eliminating a and b.

19. Solve the differential equation: $(x^2 - y^2)\,dx + 2xy\,dy = 0$

20. If $\vec{\alpha} = 3\hat{i} - \hat{j}$ and $\vec{\beta} = 2\hat{i} + \hat{j} - 3\hat{k}$. Express $\vec{\beta}$ $as\ a\ sum\ of\ two\ vectors$ $\vec{\beta}_1$ and $\vec{\beta}_2$, where $\vec{\beta}_1$ is Parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ $is\ perpendicular\ to$ $\vec{\alpha}$.

21. Find the equation of the plane through $2\hat{i} + \hat{j} - \hat{k}$ and passing through the line of intersection of the planes $\vec{r} \cdot (\hat{i} + 3\hat{j} - \hat{k}) = 0$ and $\vec{r} \cdot (\hat{j} + 2\hat{k}) = 0$.

22. In bag A there are 5 red balls and 3 white balls and in bag B there are 3 red balls and 5 white balls. If a ball is drawn from one of these two bags and found to be red, find the probability that it is drawn from bag A.

OR

A and B toss a coin alternately till one of them gets a head and wins the game. If A starts the game,
find their respective probabilities of winning.

SECTION-C

23. If A = $\begin{bmatrix} 2 & 3 & 1 \\ -3 & 2 & 1 \\ 5 & -4 & -2 \end{bmatrix}$. Find A^{-1} and hence solve the system of equations $2x - 3y + 5z = 11$; $3x + 2y - 4z = 5$ and $x + y - 2z = -3$.

24. A window is in the form of a rectangle above which there is a semi-circle. If the perimeter of the window is p cm, show that the window will allow the maximum possible light only when the radius of the semi-circle is $\frac{p}{\pi + 4}$ cm.

OR

An open box with a square base is to be made out of a given quantity of sheet of area a^2. Show that
the maximum volume of the box is $\frac{a^3}{6\sqrt{3}}$.

25. Evaluate: $\int_0^{\pi} \log(1 + \cos x)\, dx$

26. Show that the lines $\vec{r} = (5\hat{i} + 7\hat{j} - 3\hat{k}) + \lambda(4\hat{i} + 4\hat{j} - 5\hat{k})$ and $\vec{r} = (8\hat{i} + 4\hat{j} + 5\hat{k}) + \mu(7\hat{i} + \hat{j} + 3\hat{k})$ intersect. Find the point of intersection.

27. Find the area of the region enclosed between the curves $(x-1)^2+y^2 = 1$ and $x^2+y^2=1$.

OR

Find the area lying above the x-axis and included between the circle $x^2 + y^2 = 8x$ and the parabola
$y^2 = 4x$.

28. A dealer wishes to purchase a number of fans & sewing machines. He has only Rs. 5,760 to invest & has space for at most 20 items. A fan cost him Rs.360 and a sewing machine Rs. 240. He expects to gain Rs. 22 on a fan and Rs. 18 on a sewing machine. Assuming that he can sell all items he can buy, how should he invest the money in order to maximize the profit? Translate this problem mathematically and solve it.

29 If a fair coin is tossed 10 times , find the probability of (a)Exactly six heads.(b) At least six heads.
(c) At most six heads.

MODEL SAMPLE PAPER II - 2011

MATHEMATICS-XII

Time allowed: 3 hours **Max Marks: 100**

General Instructions:

f. This question paper consists of 29 questions divided into three sections A, B &C.
g. Section A consists of 10 questions each of 1 mark.

h. Section B consists of 12 questions each of 4 marks.
i. Section B consists of 7 questions each of 6 marks.
j. All questions are compulsory.

SECTION - A

1. Show that the relation in the set {1, 2, 3} given by R = {(1, 2), (2, 1)} is symmectric but not reflexive.

2. Find the principal value of $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)+\cot^{-1}\left(\cot\frac{7\pi}{6}\right)$.

3. Find the derivative of $\log_2$ (log x).

4. Find the order and the degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}=\frac{d^2y}{dx^2}$.

5. Find the value of $\hat{i}\cdot(\hat{j}\times\hat{k})+\hat{j}\cdot(\hat{k}\times\hat{i})+\hat{k}\cdot(\hat{i}\times\hat{j})$.

6. If $|\vec{a}|=2$ and $|\vec{b}|=\sqrt{3}$ and $\vec{a}\cdot\vec{b}=\sqrt{3}$, find the angle between $\vec{a}$ and $\vec{b}$.

7. If $2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix}+\begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}=\begin{bmatrix} 7 & 14 \\ 15 & 14 \end{bmatrix}$ find the value of x and y.

8. Using properties of determinants find the value of $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$.

9. Let A be a square matrix of order 3 and $|A|$= 5, then find $|4A|$.

10. Find the angle between the lines $\frac{x+4}{3}=\frac{y-1}{5}=\frac{z+3}{4}$ and $\frac{x+1}{6}=\frac{y-1}{10}=\frac{z-5}{8}$.

SECTION - B

11. Prove that the function $f(x)=\frac{2x-1}{3}, x\in R$ is one-one and onto. Also find the inverse of the function f.

12. Prove that : $\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)=\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2$.

OR

Solve for x : $\tan^{-1}\frac{x+1}{x-1}+\tan^{-1}\frac{x-1}{x}=\tan^{-1}(-7)$.

13. Using properties of determinants, prove that

$$\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = 2(x+y+z)^3$$

14. For what value of k is the given function continuous at x = 0 where

$$f(x)=\begin{cases} \frac{1-\cos 4x}{8x^2}, & x\neq 0 \\ k, & x=0 \end{cases}$$

15. If y =$x^{\cos x}$ + $(\cos x)^x$, find $\frac{dy}{dx}$.

OR

If y = $(\tan^{-1} x)^2$, show that $(x^2+1)^2 y_2 + 2x(x^2+1)y_1 = 2$.

16. Sand is pouring from a pipe at the rate of 12 cm^3/ s. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone is increasing when the height is 4 cm.

17. Evaluate $\int \frac{1}{\cos(x-a)\cos(x-b)}dx$

OR

Evaluate $\int e^{2x}\sin x \, dx$

18. Evaluate $\int_0^{\pi/4} \frac{\sin x+\cos x}{9+16\sin 2x}\, dx$

19. Evaluate the integral as limit of sum.

$$\int_0^4 (x^2+5x)\,dx$$

20. Prove that $\left|\vec{a}\times\vec{b}\right|^2 = \begin{vmatrix} \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} \\ \vec{a}\cdot\vec{b} & \vec{b}\cdot\vec{b} \end{vmatrix}$.

OR

Using vector method, find the area of the triangle whose vertices are

A(1, 1, 1), B(1, 2, 3) and C(2, 3, 1).

21. Find the shortest distance between the lines whose vector equation are

$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$ and $\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$

22. In a bolt factory, three machines A, B, C manufacture 25%, 35% and 40% of the total production respectively. Of their respective output, 5%, 4% and 2% are defective. A bolt is drawn at random from the total product and it is found to be defective. Find the probability that it was manufactured by the machine C.

SECTION - C

23. Obtain the inverse of the following matrix using elementary operations.

$$\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$$

OR

Solve the system of equations by matrix method:

$3x + 4y + 2z = 8$

$2y - 3z = 3$

$x - 2y + 6z = -2$

24. Show that the semi vertical angle of a cone of maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$.

OR

Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is 8/27 of the volume of the sphere.

25. Find the area of the region in the 1^{st} quadrant by the x-axis, the line y = x and the circle $x^2 + y^2 = 32$.

26. Solve the following differential equation:

$x\frac{dy}{dx} + 2y = x^2 \log x.$

27. Find the equation of the plane passing through the points (1, 2, 3), (0, -1, 0) and parallel to the line $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z}{-3}$.

28. Three defective bulbs are mixed with 7 good ones. Let X be the number of defective bulbs when 3 bulbs are drawn at random. Find the mean and variance of X.

29. Swati wishes to mix two types of food P and Q in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 11 units of vitamin B. Food P costs Rs. 60/ kg and food Q costs Rs. 80/ kg. Food P contains 3 units/ kg of vitamin A and 5 units/ kg of vitamin B while food Q contains 4 units/ kg of vitamin A and 2 units/ kg of vitamin B. Determine the minimum cost of the mixture.

SAMPLE PAPER -1

FOR SLOW LEARNERS ON MINIMUM CONTENT

Time-3Hrs M.M-70

General Instructions-----

1.Question paper contains 22 questions.

2.Question no. 1-8 each 1 marks, 9-19 each 4 marks and 20-22 each 6 marks.

(SECTION-A)

1. If $f(x) = x + 7$ and $g(x) = x - 7$ $x \in R$ ¸find $(fog)7$.
2. Evaluate- $\sin\left[\frac{\pi}{3} - \sin^{-1}(\frac{1}{2})\right]$
3. If A is a square matrix of order 3 and $|A| = 4$ find $|3A|$.
4. Find the value of $x \,\&\, y$ $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 2 \\ 1 & 8 \end{bmatrix} = \begin{bmatrix} 5 & 8 \\ 1 & 8 \end{bmatrix}$
5. Find the co-factor of a_{12} in the $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & 7 \end{vmatrix}$
6. Evaluate. $\int \frac{\cos\sqrt{x}}{\sqrt{x}} dx$
7. Find the unit vector in the direction of $\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$.
8. Write the vector equation of the line $\frac{x-5}{3} = \frac{y+4}{7} = \frac{6-z}{2}$

(SECTION-B)

9. Show that the relation R defined by $(a,b)R(c,d) = a + d = b + c$ on the set NxN is an equivalence relation.
10. solve for x $\sin^{-1}\frac{2a}{1+a^2} + \cos^{-1}\frac{1-b^2}{1+b^2} = 2\tan^{-1}x$

11. $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

12. Find the value of constant a & b , so that the function 'f'

defined below is continuous. $f(x) = \begin{cases} ,1 & x \le 3 \\ ax+b, & 3 < x < 5 \\ 7 & x \ge 5 \end{cases}$

13. Differentiate $x^x + (\cos x)^{\sin x}$ with respect to x.

14. Find the intervals in which the function $f(x) = \sin x + \cos x, 0 \le x \le \pi$ is strictly increasing or strictly decreasing.

15. Evaluate. $\int x^2 (\log x)\ dx$

16.Using the properties of definite integrals evaluate the integral:

$$\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$$

17.Solve the differential equation.

$\frac{dy}{dx} + y \cot x = 4x \cos ecx$, $y = 0$ when $x = \frac{\pi}{2}$

18.If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$ .Show that $\vec{a} - \vec{d}$ is parallel to $\vec{b} - \vec{c}$ where $\vec{a} \ne \vec{d}$ & $\vec{b} \ne \vec{c}$

19.Find the value of λ so that the line $\frac{x+1}{7} = \frac{y-2}{2\lambda} = \frac{z-3}{2}$ and $\frac{x-1}{3\lambda} = \frac{y-1}{1} = \frac{6-z}{7}$ are perpendicular to each other.

(SECTION-C)

20.SUsing matrices, solve the following system of equation:

$$2x - 3y + 5z = 11$$

$$3x + 2y - 4z = -5$$

$$x + y - 2z = -3$$

21. Find the area of the region bounded by the curves $y^2 = 4x$ and line $y = x$

OR

Find the inteagral as a limit of a sum of inteagral $\int_1^3 (3x^2 + 2x)dx$

22. A diet is to contain at least 80 units of vitamin A and 100 units of minerals. Two foods F_1 & F_2 are available. Food F_1 costs Rs. 4 per unit and food F_2 costs Rs.6 Per unit. One unit of food F_1 contains 3 units of vitamin A and 4 units of minerals. One unit of food F_2 contains 6 units of vitamin A and 3 units of minerals. Formulate this as a linear programming problem and find graphically the minimum cost for diet that consist of mixture of these two foods and also meets the minimal nutritional requirements.

SAMPLE PAPER -2

FOR SLOW LEARNERS ON MINIMUM CONTENT

Time-3Hrs M.M-70

General Instructions-----

1.Question paper contains 22 questions.

2.Question no. 1-8 each 1 marks, 9-19 each 4 marks and 20-22 each 6 marks.

(SECTION-A)

1. Let * be a binary operation defined by $a * b = 2a + b - 3$ find $3 * 4$.
2. Solve for x, $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$
3. Construct a 3x1 matrix $A = [a_{ij}]$ whose elements a_{ij} are given by $a_{ij} = \frac{(3i - j)^2}{2}$
4. If $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, and $B = \begin{bmatrix} 2 & -3 & 4 \end{bmatrix}$, find AB.
5. Find x if $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$
6. Evaluate: $\int \frac{\sec^2 x}{3 + \tan x} dx$
7. Write the unit vector of magnitude 15 unit in the direction of vector $\hat{i} - 2\hat{j} + 2\hat{k}$
8. If $\vec{p}$ is the unit vector and $(\vec{x} - \vec{p}) \bullet (\vec{x} + \vec{p}) = 80$ then find $|\vec{x}|$.

(SECTION-B)

9. Show that the given function $f : R \to R$ given by $f(x) = 2x$ is one-one and onto.

10. Prove. $2\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}=\frac{\pi}{4}$

11.Using the properties of determinants, prove that –

$$\begin{vmatrix} 1 & x & x^3 \\ 1 & y & y^3 \\ 1 & z & z^3 \end{vmatrix} = (x-y)(y-z)(z-x)(x+y+z)$$

12. For what value of K the function is continuous at x=2

$$f(x)=\begin{cases} 2x+1, x<2 \\ k, \quad x=2 \\ 3x-1, x>2 \end{cases}$$

13. If $\sin y = x\sin(a+y)$ then prove that $\frac{dy}{dx}=\frac{\sin^2(x+y)}{\sin a}$

14. Find the equation of the tangent to the curve $y=\sqrt{3x-2}$ which is parallel to the line $4x-2y+5=0$.

15. Evaluate. $\int x\tan^{-1}x dx$

16.Using the properties of definite integral

$$\int_o^{\pi}\frac{e^{\cos x}}{e^{\cos x}+e^{-\cos x}}dx$$

17. Solve the differential equation

$$(1+x^2)\frac{dy}{dx}+y=\tan^{-1}x$$

18.The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with the unit vector along the sum of

Vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to 1.Find the value of λ.

19. Find the shortest distance between the lines.

$\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$ and $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$

(SECTION-C)

20. If $A=\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ find A^{-1}.Using A^{-1},solve the following system of equations

$$2x-3y+5z=16$$

$$3x+2y-4z=-4$$

$$x+y-2z=-3$$

21. Find the area of the parabola $y^2=4ax$ bounded by its latus rectum.

OR

Evaluate the integral using the limit of sums.

$$\int_0^2 (x^2+x)dx$$

22. A person consumes two types of food A and B respectively to obtain 8 unit of protein. 12 units of carbohydrates and 9 units of fat.1 kg of food A contains 2,6 and 1 units of protein , carbohydrates and fat respectively and food B contains 1,1 and 3 units respectively. Food A costs Rs.8 per kg while food B costs Rs.5 per kg. Determine how many kgs of each food A and B should he buy to minimise the cost and still meets the minimal nutritional requirements.